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NATURAL CONVECTION BETWEEN CONCENTRIC SPHERES IN A SLIGHTLY-THERMALLY STRATIFIED MEDIUM

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Abstract – In the present paper, analytical solutions are obtained using perturbation expansion in powers of Grashof number for steady, axisymmetric flow of a viscous fluid contained between two concentric spheres. A uniform gravity field acts vertically downward. The outer sphere is assumed to be maintained at a variable temperature such that conditions for vertical stratification are satisfied. Analysis is presented for two cases: when a constant-heat-flux condition on the inner sphere surface is imposed or when its surface temperature is kept constant. Streamlines, isotherms and velocity components are shown graphically in an axial plane for each case. For the case of isothermal inner sphere, a dimensionless stratification parameter S governs the flow. Solutions for $S = 0$ correspond to the unstratified case. When S tends to infinity, the flow pattern has both vertical and horizontal symmetry. But when the inner sphere surface is kept at constant heat flux, the flow and temperature fields are governed by another dimensionless parameter Q. The case $Q = 0$ corresponds to thermally insulated inner sphere. For this case, flow is similar to that occurring when S tends to infinity, but the directions of the streamlines are reversed.

NOMENCLATURE

All primed quantities are dimensional; all unprimed quantities are dimensionless. Subscripted terms with m denote their corresponding values at the diametral plane ($y' = 0, \theta = \pi/2$).

- g', acceleration of gravity;
- G, square root of modified Grashof number $[g' \beta' r_i'^4 (dT'_\infty/dy')]^{1/2}/v'$;
- k', thermal conductivity;
- I_m, Gegenbauer functions of the first kind and of order m;
- Nu_i, local Nusselt number on the inner sphere;
- P_n, Legendre polynomials of the first kind and of order n;
- Q, ratio of the inner sphere constant heat flux $(\partial T'/\partial r')_{r=r_i}$ to (dT'_∞/dy');
- r', r, radial coordinate $r = r'/r_i'$;
- r_i', r_0', radii of the inner and the outer spheres;
- R, ratio r_0'/r_i';
- S, steepness parameter defined as $\{r_d'(dT'_\infty/dy')\}$ divided by the temperature difference between inner sphere and the fluid occupying the diametral plane;
- T', T, temperature $T' = T_m' + r_i'(dT'_\infty/dy')T$;
- dT'_∞/dy', constant temperature gradient describing the constant stratification;
- v_r', v_r, r-component of velocity $v_r = v_r' r_i'/v'G$;
- v_θ', v_θ, θ-component of velocity $v_\theta = v_\theta' r_i'/v'G$;
- y', y, $y' = r' \cos\theta$, $y = y'/r_i' = r' \cos\theta/r_i'$.

Greek symbols

- α', thermal diffusivity;
- β', volumetric coefficient of thermal expansion;
- θ, colatitude or polar angle measured from the upward vertical $\theta = 0$;
- ρ', density;
- v', kinematic viscosity;
- ψ', ψ, Stream function $\psi = \psi'/Gv'$.

1. INTRODUCTION

FREE convection heat transfer in spherical annulus has been the subject of many investigations. Bishop, Kolflat *et al.* [1] were first to present the flow visualization studies depicting three different convective flow patterns of the fluid (air) contained between two isothermal concentric spheres: two steady patterns, the crescent eddy and the kidney-shaped eddy types, and one unsteady pattern, falling vortices type. These flow patterns depended on the low-to-large-diameter ratio of the spheres and moderate to large temperature differences. In additional papers [2–4], measured temperature profiles were analyzed and overall heat-transfer rates were correlated. Yin, Powe *et al.* [5] performed experiments concerning natural convection between two concentric spheres, the inner one being hotter. The convecting fluids were air and water. Observed flow patterns were correlated with previously published temperature profiles and were categorized in terms of steady and unsteady regimes. The results of a flow visualization study of natural convection in air between a heated sphere and its cooled cubical enclosure were reported by Powe, Scanlan and Eyler [6]. Mack and Hardee [7] calculated the first three terms of the perturbation solution for natural convection between concentric spheres in powers of Raleigh numbers. Streamlines, velocity and temperature distributions were presented for Raleigh number equal to 1000 and Prandtl number P equal to 0.7.

Experiments on natural convection from isothermal spheres and cylinders immersed in a thermally stratified fluid were performed by Eichhorn, Lienhard and Chen [8]. Heat-transfer results and visual observations of the flow field were presented for various values of the steepness parameters S. Hubbel and Gebhart [9] made observations on convective transport and plume shedding induced by a heated horizontal cylinder submerged in quiescent, salt-stratified water. Chen and Eichhorn [10] studied both analytically and experimentally natural convection from an isothermal finite plate immersed in a stable thermally stratified fluid. Local and overall heat-transfer coefficients, velocity and temperature profiles were given for Prandtl number equal to 6. Natural convection problems from simple bodies immersed in thermally stratified fluids have recently been reviewed in a report prepared by Chen and Eichhorn [11]. They gave a design for an enclosure to produce quickly and reliably a thermally stratified environment, a problem which is not as simple as it first appears to be. In the report overall heat-transfer rates and a limited study of the behavior of thermal plumes from immersed horizontal cylinders and spheres in stratified fluids were presented. Results based on an approximate boundary layer analysis using local nonsimilarity and series solution methods compared reasonably well with those of experiments. Singh [12] obtained analytical solutions for axisymmetric flow of a vertically stratified viscous fluid by the singular perturbation technique valid for small Grashof numbers.

Free convection between horizontal concentric cylinders was considered by Singh and Elliott [13] when the outer cylinder is assumed to be maintained at a variable temperature such that conditions for vertical stratifications are satisfied. The inner cylinder is either thermally insulated or its surface temperature is kept constant. Theoretical solutions were obtained in power series of Grashof number G and streamlines, isotherms and velocity profiles were plotted for various values of the steepness parameter S. In this paper the perturbation solution is extended for the free convection problem between two concentric spheres. Isotherms, streamlines and velocity components are shown graphically in an axial plane for various values of the two dimensionless parameters S and Q and for $P = 0.7$, $G = 2$, the radius ratio $R = 2$. When the inner sphere surface is kept at a uniform temperature, details of the fluid motion are dependent on the steepness parameter S. Local Nusselt numbers on the inner sphere Nu_i are calculated and the ratio $(Nu_i/Nu_{s=0})$ is plotted vs. θ for various values of S. In the case of the inner sphere maintained at constant heat flux, the parameter Q governs the flow. Streamlines and velocity components, which are qualitatively similar to those in concentric horizontal cylinders [13], are also analagous for various values of both parameters S and Q', but the flow directions are reversed.

2. MATHEMATICAL FORMULATION AND PERTURBATION SOLUTION

A viscous, incompressible fluid occupies the region between two concentric spheres of radii r_i' and r_0'. The flow is symmetrical about a vertical diameter which is taken as the axis $\theta = 0$ of spherical polar coordinates (r', θ, ϕ) with the origin at the center of the spheres. A uniform gravity field is acting vertically downward and hence all quantities are independent of ϕ. The inner sphere is either kept at a constant temperature T_i' or is maintained at constant heat flux. The outer sphere surface is maintained at a variable temperature T_0' such that vertical stratification is satisfied, i.e.:

$$T_0' = T_m' + (\mathrm{d}T_\infty'/\mathrm{d}y')r_0' \cos\theta, \quad y = r'\cos\theta. \quad (1)$$

The velocity components are related to stream function ψ as given by:

$$v_r = \frac{\partial\psi/\partial\theta}{r^2 \sin\theta}, \quad v_\theta = -\frac{\partial\psi/\partial r}{r\sin\theta}. \quad (2)$$

Introducing the Boussinesq approximation [14]

$$\frac{\rho' - \rho_m'}{\rho_m'} = -\beta(T' - T_m'),$$

$$T' = T_m' + (\mathrm{d}T_\infty'/\mathrm{d}y')r_i' T \quad (3)$$

into the Navier–Stokes equations, we get for the steady, axisymmetric motion [7, 12]:

$$D^4\psi = -Gr\sin\theta\left(\sin\theta\frac{\partial T}{\partial r} + \frac{\cos\theta}{r}\frac{\partial T}{\partial\theta}\right)$$
$$+ G\sin\theta\left(\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r} - \frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\right)\left(\frac{D^2\psi}{r^2\sin^2\theta}\right) \quad (4)$$

$$\nabla^2 T = \frac{PG}{r^2\sin\theta}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial T}{\partial r} - \frac{\partial\psi}{\partial r}\frac{\partial T}{\partial\theta}\right) \quad (5)$$

where

$$D^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$$

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\frac{\partial}{\partial\theta}\right).$$

The boundary conditions in view of (1) and (3) are:

$$\psi = \frac{\partial\psi}{\partial r} = 0, \; T = \frac{1}{S} \text{ or } \frac{\partial T}{\partial r} = Q \text{ at } r = 1; \quad (6)$$

$$\psi = \frac{\partial\psi}{\partial r} = 0, \; T = R\cos\theta \text{ at } r = R. \quad (7)$$

For a given radius ratio R, the solution of (4) and (5) subject to conditions (6) and (7) depends on three parameters, P, G, and S or Q. It is obtained in power series of G when P and S or Q are assumed fixed

$$\psi = \psi_0(r,\theta) + G\psi_1(r,\theta) + G^2\psi_2(r,\theta) + \ldots \quad (8)$$

$$T = T_0(r,\theta) + GT_1(r,\theta) + G^2T_2(r,\theta) + \ldots \quad (9)$$

Substitution of (8) and (9) gives for the zeroth power of G:

$$D^4\psi_0 = 0, \nabla^2 T_0 = 0. \tag{10}$$

Since ψ and $\partial\psi/\partial r$ both vanish at $r = 1$ and R, ψ_0 is zero throughout. Similarly $T_1, T_3 \ldots$ and $\psi_2, \psi_4 \ldots$ are zero. The solution for T_0 satisfying (6) and (7) is:

$$T_0 = \frac{1}{S(R-1)}\left(\frac{R}{r} - 1\right) + \frac{R^3 \cos\theta}{(R^3-1)}\left(r - \frac{1}{r^2}\right) \tag{11}$$

or

$$T_0 = \frac{Q}{R}\left(1 - \frac{R}{r}\right) + \frac{R^3 \cos\theta}{(R^3+0.5)}\left(r + \frac{1}{2r^2}\right). \tag{12}$$

Equations (11) and (12) represent the solution for the isothermal and constant heat flux inner sphere cases, respectively. For the next approximation, when (8) and (9) are substituted into (4) and (5), we get:

$$D^4\psi_1 = \frac{R\sin^2\theta}{S(R-1)r} - \frac{3R^3\sin^2\theta\cos\theta}{(R^3-1)r^2}, \tag{13}$$

and

$$D^4\psi_1 = \frac{-Q\sin^2\theta}{r} + \frac{3R^3\sin^2\theta\cos\theta}{(2R^3+1)r^2}. \tag{14}$$

The solution of (13) or (14) satisfying $\psi_1 = \partial\psi_1/\partial r = 0$, both $r = 1$ and $r = R$, is:

$$\psi_1 = (B_1 r^4 + D_1 r^3 + B_2 r^2 + B_3 r + B_4/r)\sin^2\theta + (C_1 r^5 + C_2 r^3 + C_0 r^2 + C_3 + C_4/r^2)\sin^2\theta\cos\theta \tag{15}$$

where

$$B_1 = D_1(2R^7 - 6R^6 + 4R^5 + 4R^4 - 6R^3 + 2R^2)/B_0$$

$$B_2 = D_1(2R^9 - 12R^7 + 10R^6 + 10R^5 - 12R^4 + 2R^2)/B_0$$

$$B_3 = D_1(-3R^9 + 8R^8 - 5R^7 - 5R^5 + 8R^4 - 3R^3)/B_0$$

$$B_4 = D_1(R^9 - 4R^8 + 6R^7 - 4R^6 + R^5)/B_0$$

$$B_0 = -4R^8 + 9R^7 - 10R^5 + 9R^3 - 4R^2$$

$D_1 = -R/8S(R-1)$ for isothermal inner sphere

$= Q/8$ for constant heat flux

and

$$C_0 = \delta R^3/8(R^3+\delta)$$

$$C_1 = C_0(R^7 - 6R^5 + 5R^4 + 5R^3 - 6R^2 + 1)/C_5$$

$$C_2 = C_0(-3R^9 + 10R^7 - 7R^4 + 10R^2 - 3)/C_5$$

$$C_3 = 2C_0 - 3.5C_1 - 2.5C_2$$

$$C_4 = 2.5C_1 + 1.5C_2 + C_0$$

$$C_5 = 2R^{10} - 12.5R^7 + 21R^5 - 12.5R^3 + 2$$

and δ is equal to -1 when the inner sphere temperature is constant and 0.5 when the sphere is at constant heat flux.

Similarly, expressions for T_2 and ψ_3 can be obtained by making use of (4), (5), (11), (12), and (15) as given by the following

$$T_2(r,\theta) = \sum_{n=1}^{3} f_n(r) P_n(\cos\theta)$$

$$\psi_3(r,\theta) = \sum_{m=1}^{4} F_m(r) I_m(\cos\theta)$$

where $P_n(\cos\theta)$ and $I_m(\cos\theta)$ are the Legendre polynomials and Gegenbauer functions of the first kind and of orders n and m, respectively [15]. The coefficients f_n and F_m are functions of r, R, P and Q, and are very long. These are omitted to conserve space; readers interested in them are invited to write to the authors.

3. DISCUSSION OF RESULTS

First of all, we attempt to find what is the maximum value of G called G_{max} for which the two-terms expansion solution obtained in powers of G is con-

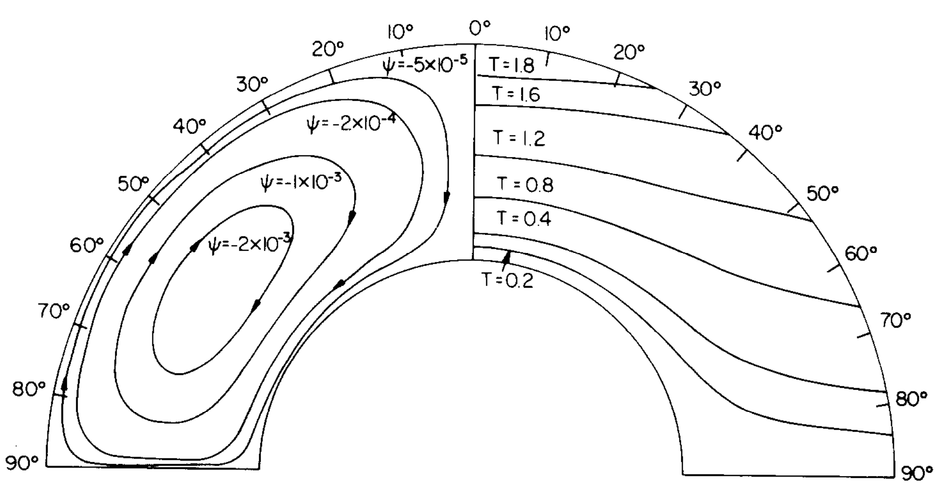

FIG. 1. Streamlines and isotherms for $S = \infty$, $G = 2$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 54.5$ and 125.5°.

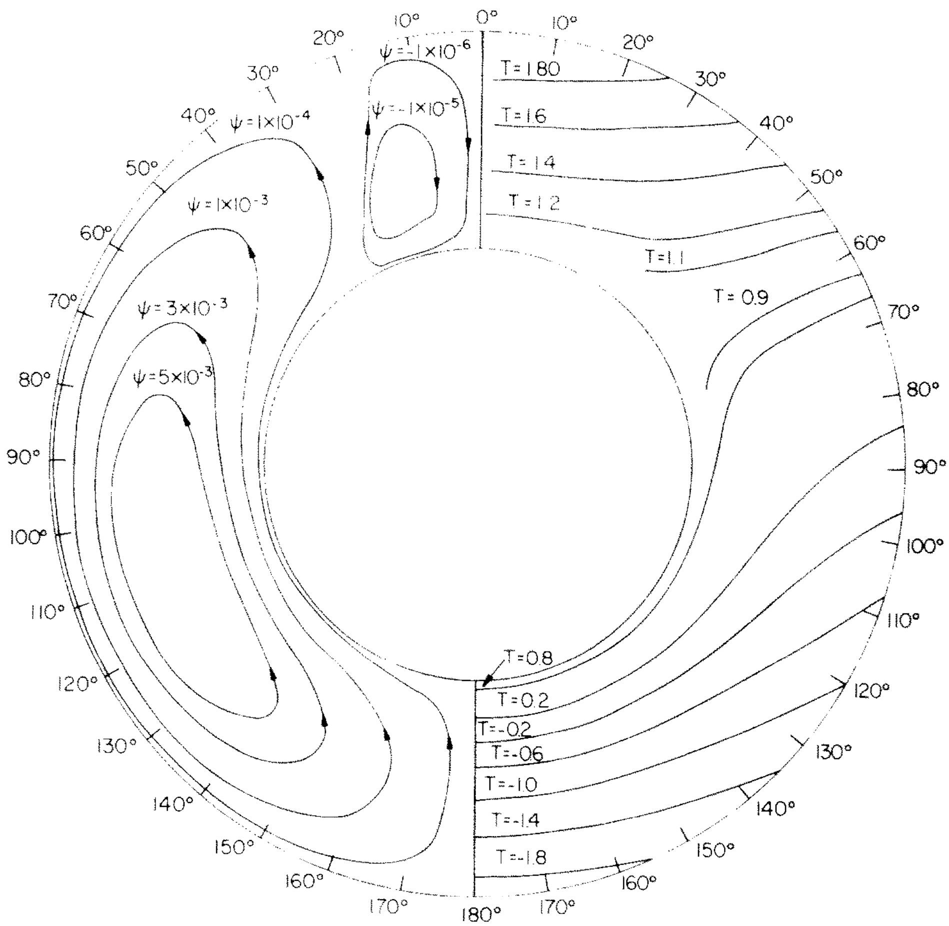

FIG. 2(a). Streamlines and isotherms for $S = 1$, $G = 2$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 18$ and 110°.

vergent for $R = 2$, $P = 0.7$ and various values of S and Q of order unity. As a crude measure of the upper bound for convergence Mack and Hardee ([7], §3.1) defined G_{max} as that value of Grashof number for which the maximum (with respect to position) magnitude of any higher-order term in either series for T and ψ equals the maximum magnitude of the appropriate lowest-order term T_0 or $G\psi_1$. The maximum magnitude of r-function coefficients of $\sin^2\theta$ and $\sin^2\theta\cos\theta$ in (15) are of the order of 10^{-2}, whereas the largest values of the various coefficients of $P_n(\cos\theta)$ and $I_m(\cos\theta)$ in the expressions for $T_2(r,\theta)$ and $\psi_3(r,\theta)$ are of the order of 10^{-3} and 10^{-4} respectively. Hence, according to the above-mentioned criterion we find that the two-term perturbation solution converges for $G \leq 5$.

Behavior of streamlines and isotherms in an axial plane is described in detail for fixed values of $R(=2)$, $G(=2)$, and $P(=0.7)$ for many values of S ranging from zero to infinity. For the steepness parameter S tending to zero, constant stratification (dT'_∞/dy) vanishes and the limit of the solution approaches the unstratified case of Mack and Hardee [7]. For this case when the inner sphere surface is kept at a temperature higher than that of the outer sphere, streamlines consist of single cells of 'crescent eddy' type. The flow is symmetrical about the vertical diameter $\theta = 0, \pi$ and is upward along the inner sphere (counter clockwise) and downward along the outer sphere (clockwise). But when S tends to infinitely large values, T'_i equals T'_m, i.e. the inner sphere temperature is equal to that of the diametral plane $y' = 0 (\theta = \pi/2)$ of the outer sphere. Motion in this case is symmetrical about both the horizontal ($\theta = \pi/2$) and the vertical ($\theta = 0, \pi$) diametral planes. The expression for the stream function ψ_1 (15) (the dominant term) becomes, since $D_1 = 0$ for $S = \infty$:

$$\psi_1 = \sin^2\theta \cos\theta \, (C_1 r^5 + C_2 r^3 + C_0 r^2 + C_3 + C_4/r^2). \quad (16)$$

The radial velocity component v_r for this case is given by [using (2)]

$$v_r = \frac{1}{r^2 \sin\theta} \frac{\partial\psi}{\partial\theta} = \frac{(3\cos^2\theta - 1)}{r^2} \times (C_1 r^5 + C_2 r^3 + C_0 r^2 + C_3 + C_4/r^2). \quad (17)$$

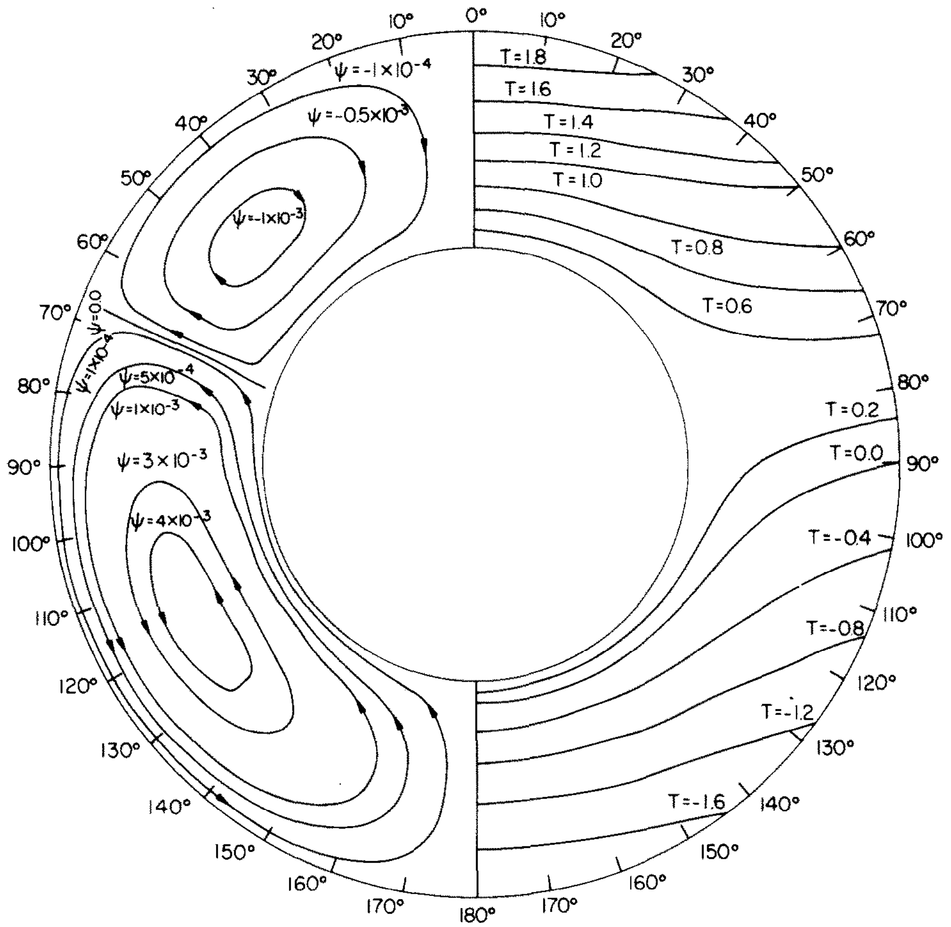

FIG. 2(b). Streamlines and isotherms for $S = 2.5, G = 2, P = 0.7, R = 2$. Radial velocity changes sign at $\theta =$ 45 and 118°.

Streamlines for this case are shown graphically in Fig. 1. Since $\delta = -1$, v_r is negative at $\theta = 0$, positive at $\theta = \pi/2$ and vanished at $\theta = 54.5°$. An inflow at the poles $\theta = 0$ from the outer sphere to the inner sphere reverses into an outflow at the equator $\theta = \pi/2$, transition taking place at $\theta = 54.5°$. Eichhorn *et al.* [8] observed a similar streamline picture for $S = \infty$ in their flow visualization for a sphere in a stratified medium. In case of two concentric horizontal cylinders, when the outer cylinder is maintained at a variable temperature of a constant stratification type such as presented in this paper, Singh and Elliott [13] find similar streamline behavior for $S = \infty$.

For finite values of the steepness parameters $S(0 < S < \infty)$, streamlines depend on the basis of superposition of the two above-mentioned flows for $S = 0$ and $S = \infty$. When S is very small, motion in an axial plane is symmetrical in the two halves of the annulus like that of the unstratified case. But as S is increased (perhaps to a value of 0.9), the single cell flow changes into a double cell flow and a region of reversed flow occurs near $\theta = 0$. A further increase in the value of S moves the angle of separation of one cell from the other toward $\theta = \pi/2$.

Figures 2(a), (b), (c) show plots of the streamlines for fixed values of $R = 2, G = 2, P = 0.7$ and for $S = 1, 2.5$ and 10, respectively. These graphs depict the motion discussed above. The velocity components v_r and v_θ are shown graphically in an axial plane vs. radial position for various values of θ in Figs. 3 and 4. Radial velocity v_r vanishes and changes sign at $\theta = 18°$, and 110° for $S = 1$, at $\theta = 45°$ and 118° for $S = 2.5$ and at $\theta = 53°$ and 123° for $S = 10$. Meridianal velocity component v_θ becomes zero and reverses its sign at $\theta = 20°$ for $S = 1$, at $\theta = 68°$ for $S = 2.5$ and at $\theta \leqq 85°$ for $S = 10$. The local Nusselt number on the inner sphere is defined as:

$$Nu = -\frac{R-1}{R}\left[r^2\frac{\partial T}{\partial r}\right]_{r=1}.$$

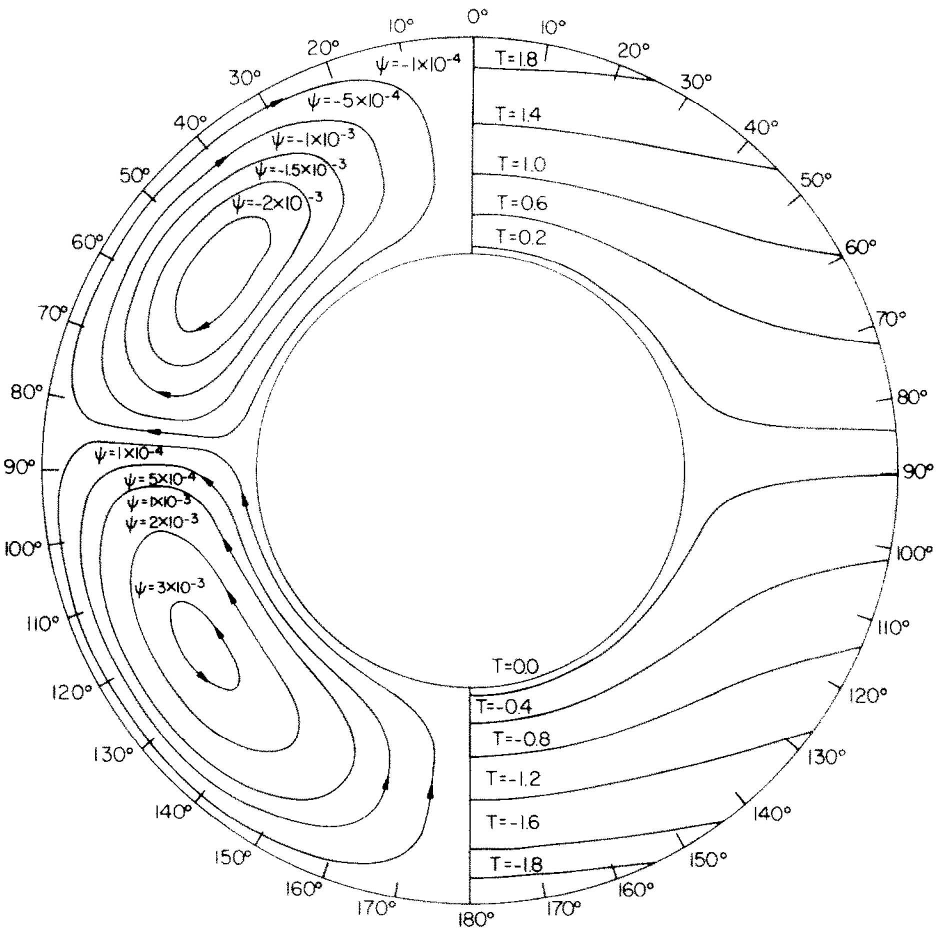

FIG. 2(c). Streamlines and isotherms for $S = 10, G = 2, P = 0.7, R = 2$. Radial velocity changes sign at $\theta = 53$ and $123°$.

For $S = 0$, local Nusselt number for the inner cylinder $Nu_{s=0}$ can be obtained from the calculations of Mack and Hardee [7]. In Fig. 5, is shown the plot of $(Nu/Nu_{s=0})$ vs. θ for $G = 2$ and for various values of S. It is found that Nu decreases with S.

When the inner sphere is maintained at constant heat flux, streamlines and isotherms depend on Q. The case $Q = 0$ corresponds to the inner sphere surface being thermally insulated. For this case, $D_1 = 0, \delta = 0.5$ and from (12), (14) and (15) one finds that all the qualitative features of the flow will be the same as for $S = \infty$, but the directions of the flow will be reversed. Figure 6 shows the sketch of streamlines and isotherms for this case. For finite values of Q, flow patterns, radial and azimuthal velocity components are shown in Figs. 7–10. These are similar to those of finite values of S, but the flow is in opposite direction.

For large values of Q, the flow pattern consists of single cells of 'crescent eddy' type and is similar to that for $S = 0$. But the flow lines are downward along the inner sphere (counter clockwise) and upward along the outer (clockwise). Isotherms are also shown graphically in Figs. 7 and 8. These are quite different from those in which the inner sphere is kept at constant temperature.

Although the detailed solution curves for flow pattern and temperature distribution are presented in this paper for a small value of the Grashof number equal to two, $P = 0.7$ and $R = 2$, the influence of variation of S and Q on the plume formation on a body in a thermally-stratified medium is brought out clearly. The plume formations on an isothermal inner sphere are in qualitative agreement with those reported by Eichhorn, Lienhard and Chen [8]. But when the inner sphere is maintained at constant heat flux, the plume-formation analysis has not been experimentally presented in the literature and the results obtained in this paper seem to be new.

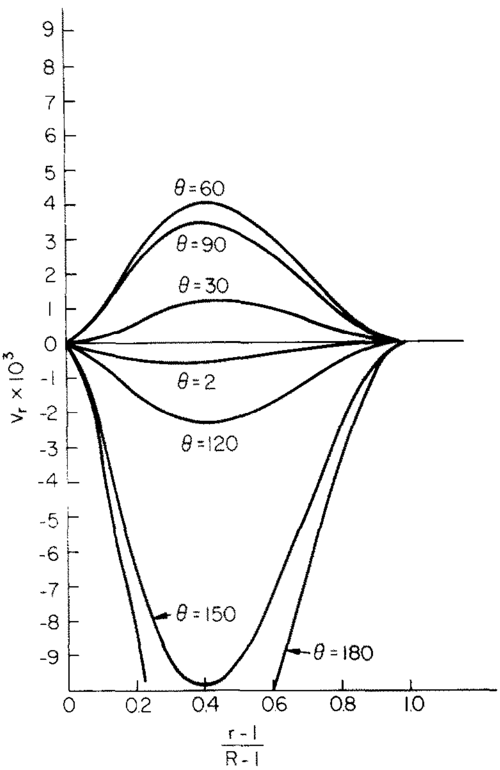

FIG. 3(a). Radial component of velocity vs. radial position for $S = 1, G = 2, P = 0.7, R = 2$. v_r vanishes and changes sign at $\theta = 18$ and 110°.

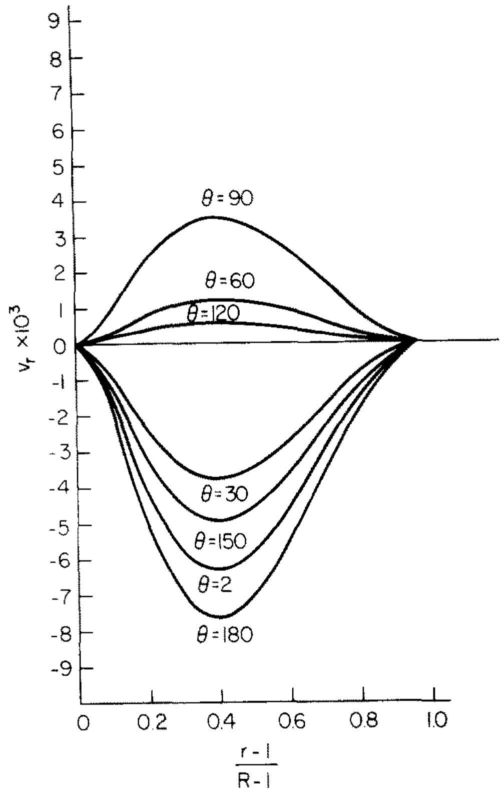

FIG. 3(c). Radial component of velocity vs. radial position for $S = 10, G = 2, P = 0.7, R = 2$. v_r vanishes and changes sign at $\theta = 53$ and 123°.

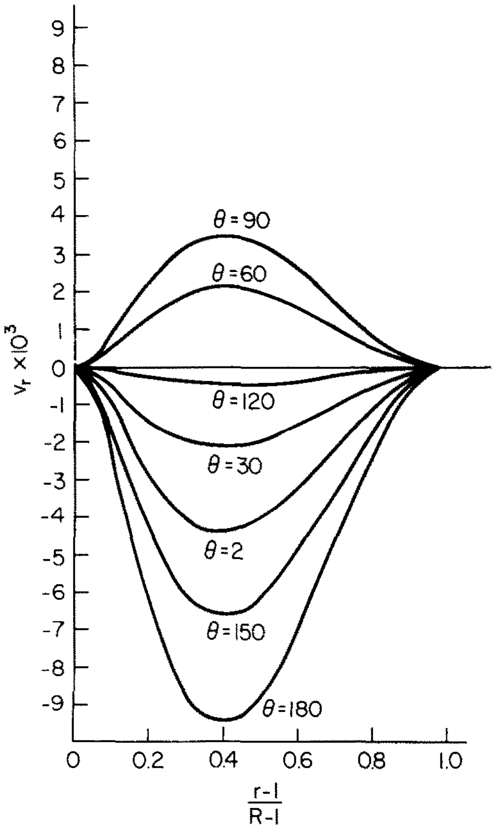

FIG. 3(b). Radial component of velocity vs. radial position for $S = 2.5, G = 2, P = 0.7, R = 2$. v_r vanishes and changes sign at $\theta = 45$ and 118°.

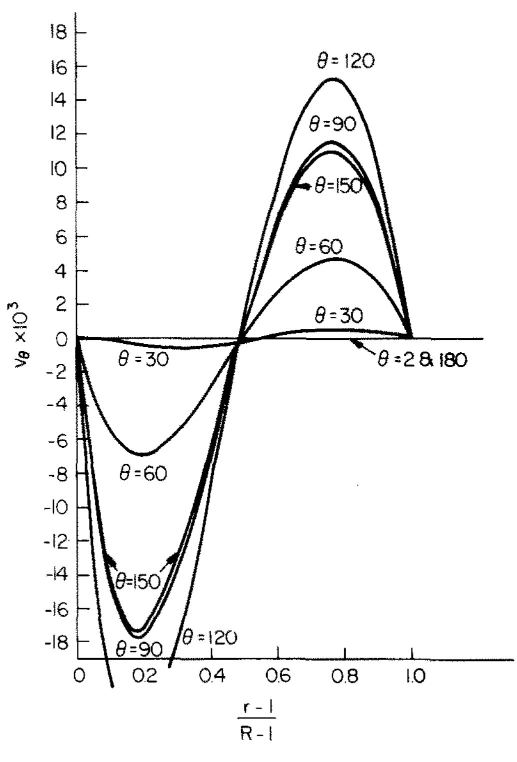

FIG. 4(a). Theta component of velocity vs. radial position for $S = 1, G = 2, P = 0.7, R = 2$. v_θ vanishes and changes sign at $\theta = 20°$.

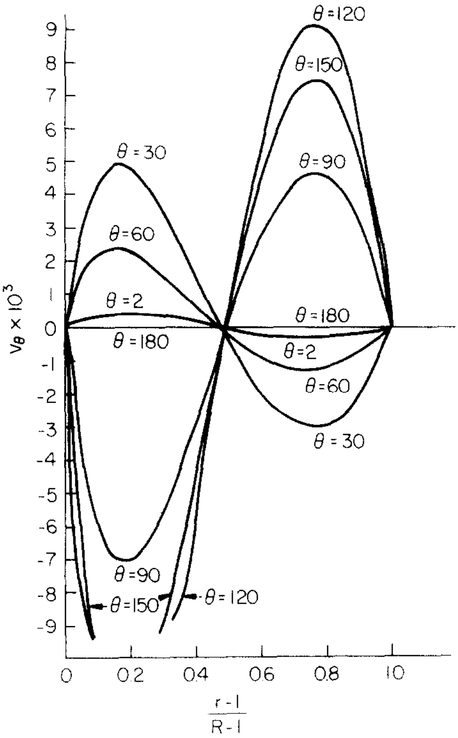

FIG. 4(b). Theta component of velocity vs. radial position for $S = 2.5, G = 2, P = 0.7, R = 2$. v_θ vanishes and changes sign at $\theta = 68°$.

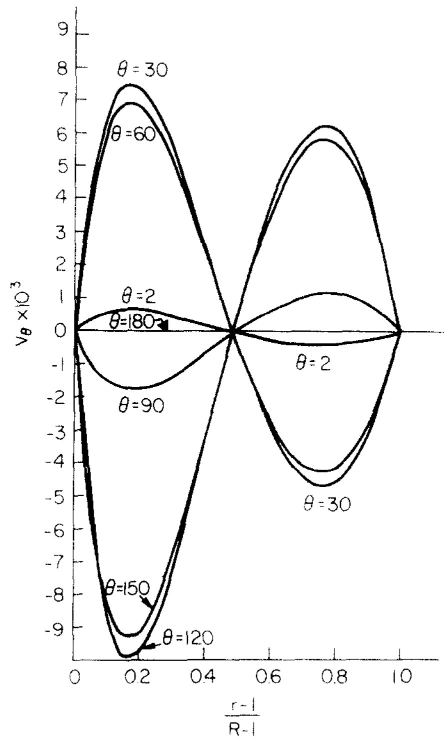

FIG. 4(c). Theta component of velocity vs. radial position for $S = 10, G = 2, P = 0.7, R = 2$. v_θ vanishes and changes sign at $\theta = 85°$.

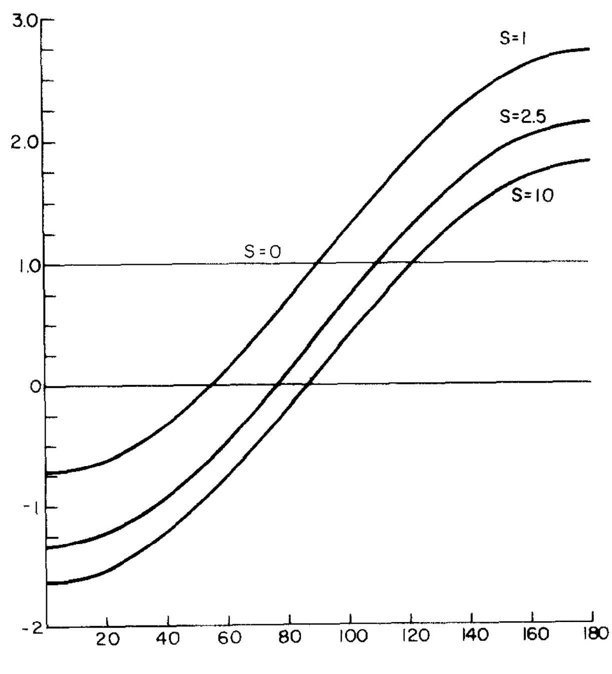

FIG. 5. Ratio of local Nusselt number for the inner sphere to the Nusselt number for $S = 0$ vs. angular position.

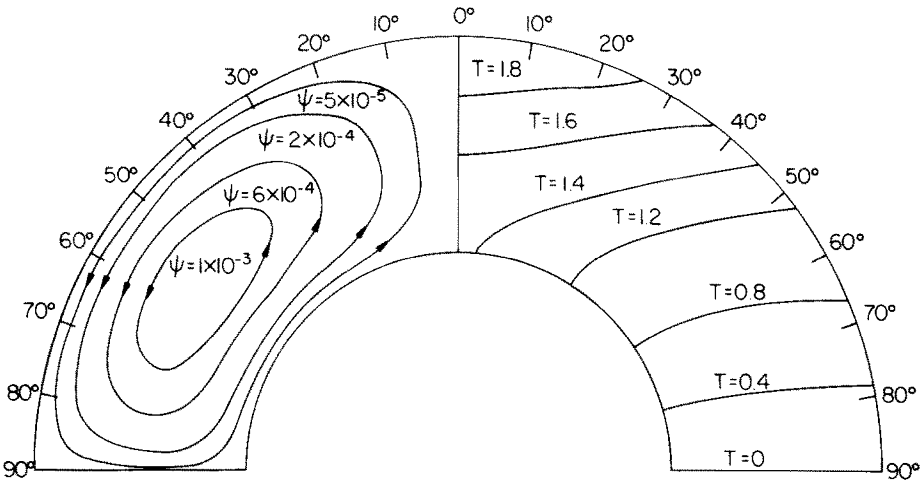

FIG. 6. Streamlines and isotherms for the case with the inner sphere thermally insulated. $S = 0$.

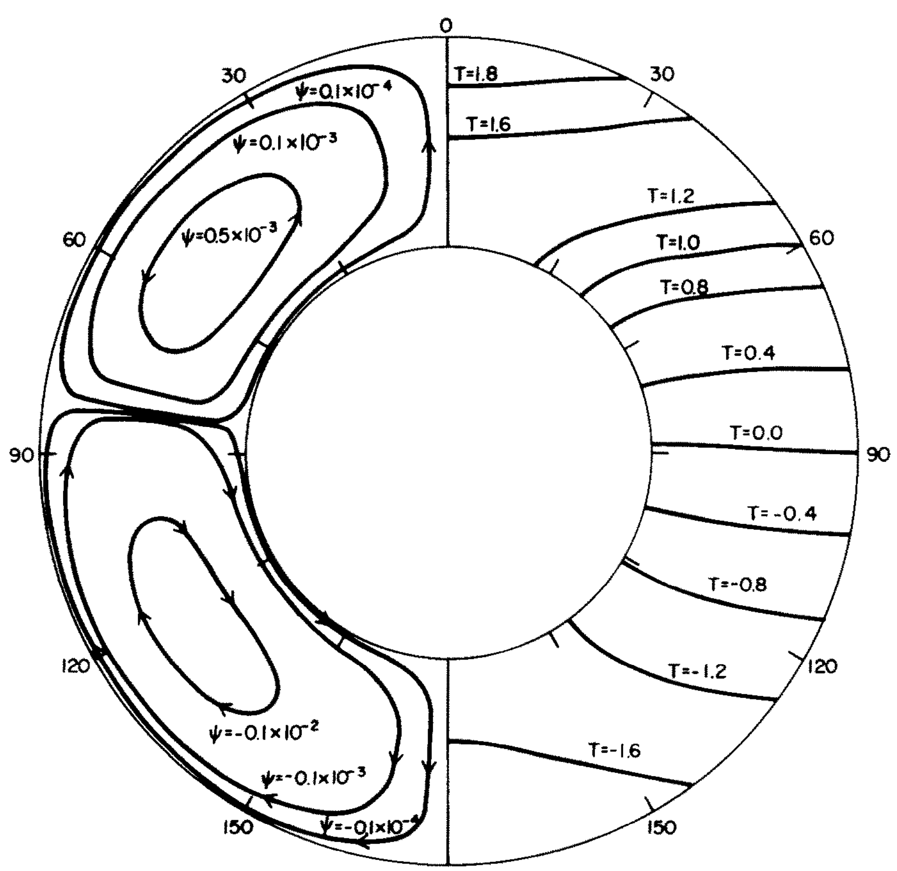

FIG. 7. Streamlines and isotherms for $Q = 0.1$, $G = 2$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 51$ and 123°.

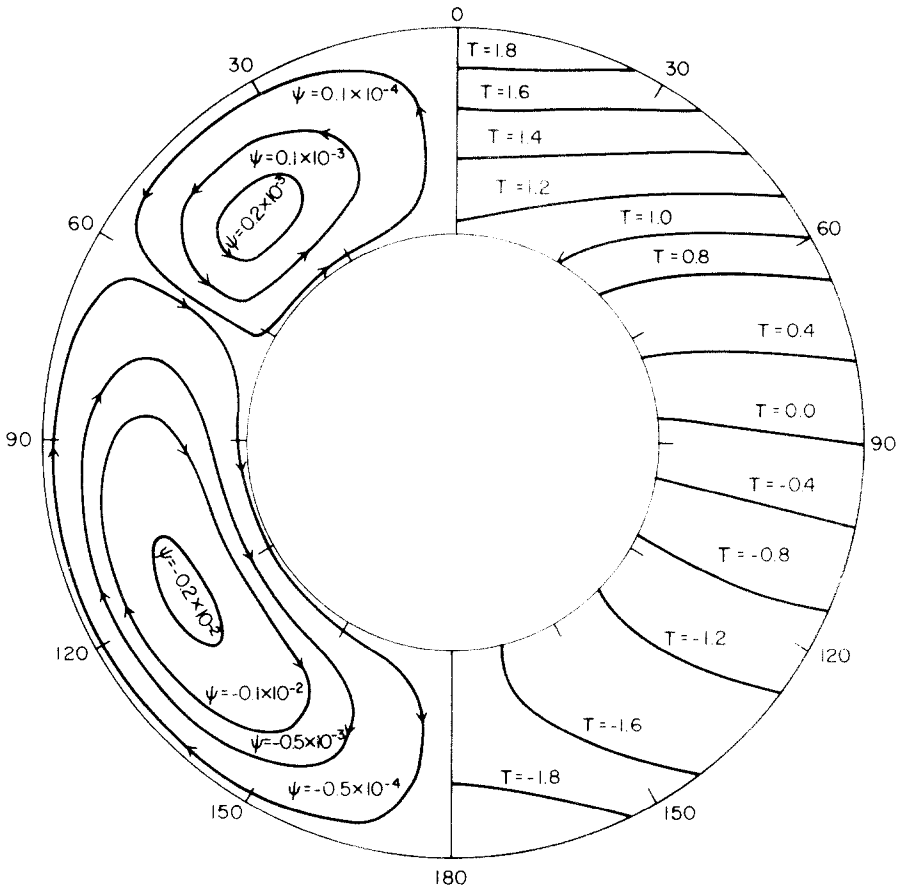

FIG. 8. Streamlines and isotherms for $Q = 0.3$, $G = 2$, $P = 0.7$, $K = 2$. Radial velocity changes sign at $\theta = 41$ and 119°.

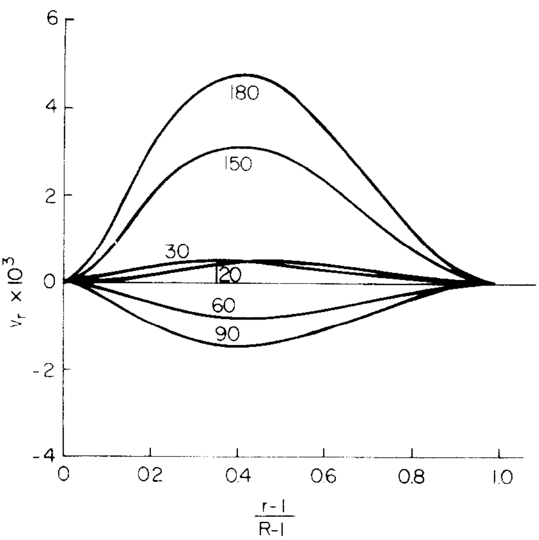

FIG. 9. Radial component of velocity vs. radial position for $Q = 0.3$, $G = 2$, $P = 0.7$, $R = 2$. v_r vanishes and changes sign at $\theta = 41$ and 119°.

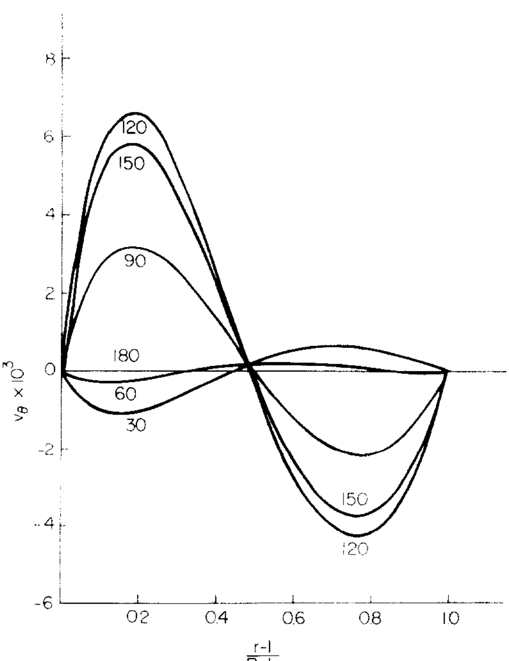

FIG. 10. Theta component of velocity vs. radial position for $Q = 0.3$, $G = 2$, $P = 0.7$, $R = 2$. v_θ vanishes and changes sign at 62°.

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CONVECTION NATURELLE ENTRE DES SPHERES CONCENTRIQUES DANS UN MILIEU LEGEREMENT STRATIFIE THERMIQUEMENT

Résumé—On obtient des solutions analytiques en développement de perturbation en puissances du nombre de Grashof pour un écoulement permanent et axisymétrique de fluide visqueux entre deux sphères concentriques. Un champ de gravité agit verticalement vers le bas. La sphère extérieure est maintenue à une température variable de telle sorte que la stratification verticale est réalisée. L'analyse est présentée dans deux cas: lorsqu'une condition de flux constant est imposée sur la sphère intérieure et lorsque la température de surface est maintenue constante. Dans chaque cas, on montre les lignes de courant, les isothermes et les vitesses dans un plan méridien. Dans le cas de la sphère intérieure isotherme, un paramètre adimensionnel de stratification S gouverne l'écoulement. Des solutions pour $S = 0$ correspondent au cas sans stratification. Quand S tend vers l'infini, la configuration de l'écoulement possède une symétrie à la fois verticale et horizontale. Mais quand la sphère intérieure est maintenue à flux constant, l'écoulement et le champ de température sont gouvernés par un autre paramètre adimensionnel Q. Le cas $Q = 0$ correspond à la sphère interne isolée thermiquement. Dans ce cas, l'écoulement est semblable à celui où S tend vers l'infini, mais les directions des lignes de courant sont inversées.

FREIE KONVEKTION ZWISCHEN KONZENTRISCHEN KUGELN IN EINEM MEDIUM MIT SCHWACHER THERMISCHER SCHICHTUNG

Zusammenfassung — In der vorliegenden Arbeit werden unter Verwendung von Störgliedansätzen in Potenzen der Grashof-Zahl analytische Lösungen für stationäre, achsensymmetrische Strömung eines viskosen Fluids erhalten, das zwischen konzentrischen Kugeln eingeschlossen ist. Ein gleichmäßiges Gravitationsfeld wirkt vertikal nach unten. Für die äußere Kugel ist angenommen, daß sie auf variabler Temperatur gehalten wird, so daß die Bedingungen für vertikale Schichtung erfüllt sind. Die Untersuchung wird für zwei Fälle durchgeführt: es wird eine konstante Wärmestromdichte an der Oberfläche der inneren Kugel aufgeprägt, oder deren Oberflächentemperatur wird konstant gehalten. Stromlinien, Isothermen und Geschwindigkeitskomponenten werden für jeden Fall in einer axialen Ebene grafisch dargestellt. Im Fall der isothermen inneren Kugel beherrscht ein dimensionsloser Schichtungsparameter S die Strömung. Lösungen für $S = 0$ entsprechen dem nicht geschichteten Fall. Wenn S gegen unendlich geht, hat der Strömungszustand sowohl vertikale als auch horizontale Symmetrie. Wenn jedoch auf der inneren Kugeloberfläche konstante Wärmestromdichte herrscht, werden die Strömungs- und Temperaturfelder durch einen anderen dimensionslosen Parameter Q bestimmt. Der Fall $Q = 0$ entspricht der thermisch isolierten inneren Kugel. In diesem Fall ist die Strömung ähnlich derjenigen, die sich einstellt, wenn S gegen unendlich geht, aber die Richtungen der Stromlinien sind umgekehrt.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ МЕЖДУ КОНЦЕНТРИЧЕСКИМИ СФЕРАМИ В СРЕДЕ СО СЛАБОЙ ТЕРМИЧЕСКОЙ СТАТИФИКАЦИЕЙ

Аннотация — Разложением в ряд по степеням числа Грасгофа получены аналитические решения для стационарного осесимметричного течения вязкой жидкости между двумя концентрическими сферами. Однородное гравитационное поле направлено вертикально вниз. Предполагается, что переменной является температура внешней сферы, так что создаются условия для стратификации в вертикальном направлении. Анализируются два случая: постоянный тепловой поток, подводимый к поверхности внутренней сферы, и постоянная температура поверхности этой сферы. Для каждого случая дано графическое изображение линий тока, изотерм и компонент скорости на осевой плоскости. Для изотермической внутренней сферы течение контролируется параметром стратификации. Решение для $S = 0$ соответствует отсутствию стратификации. При S, стремящемся к бесконечности, течение характеризуется как вертикальной, так и горизонтальной симметрией. Однако при постоянстве теплового потока на внутренней сфере гидродинамический и температурный режимы контролируются другим безразмерным параметром (Q). Равенство $Q = 0$ соответствует случаю термически изолированной внутренней сферы. Здесь картина течения аналогична той, которая наблюдается при $S \to \infty$, но с противоположным направлением линий тока.